An Empirical Examination of Volatility on Intraday Nikkei 225 Futures: A Bayesian Approach (Financial Modeling and Analysis)

Ochiai, Natsumi; Ohnishi, Masamitsu

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An Empirical Examination of Volatility on Intraday Nikkei 225 Futures: A Bayesian Approach

Natsumi Ochiai
Graduate School of Economics
Osaka University

Masamitsu Ohnishi
Graduate School of Economics & Center for Mathematical Modeling and Data Science
Osaka University

1 Introduction

A globalization in capital markets and the development of technology have brought a highly–advanced financial markets. These facts have increased the risk for the global, immediate spillover effects on volatilities in stock market. In last financial crisis, major markets got into a global–financial panic, and the aftershocks lasted for a long period from mid 2007 to 2010. We can observe the spread of such global price volatilities as a change of intraday volatilities in stock market.

Intraday volatilities have different features with daily volatilities. Generally, it is known that intraday volatilities have a U–shaped pattern during trading hours. Owing to the feature of intraday volatilities, we cannot make use of the method of time series for daily volatility straightforwardly. So far, there is some literatures on the intraday volatility models, see Andersen and Bollerslev [1, 2], and Engle and Sokalska [4]. These researches have basically used ARCH–type models. By contrast, Stroud and Johannes [7] has proposed a Bayesian modeling for volatility of high–frequency return. They have modeled intraday volatilities for 5–min returns by four components, and sampled the parameters for each component from a posterior density using Markov Chain Monte Carlo (MCMC) method.

In this article, we introduce a theoretical framework of the intraday volatility modeling which is proposed in Stroud and Johannes [7] in order to prepare for an estimation of Nikkei 225 Futures. In the next step, we would like to analyze the following: (i) we compare the empirical results for Nikkei 225 Futures with those of previous studies, because there is little empirical studies about intraday volatilitis in Japan, (ii) we examine the specific factors affecting intraday volatilities in Japan, (iii) In the light of data of last financial crisis, we test the characteristics of volatilities in financial panic.

This article is organized as follows. Section 2 and 3 present the model and the estimation algorithm proposed in Stroud and Johannes [7], respectively. Concluding remarks are contained in Section 4.

2 Stochastic Volatility Models

We consider a SV (Stochastic Volatility) model for 5–min intraday financial returns. Intraday logarithmic price returns are described by

$$y_t = 100 \cdot \log\left(\frac{P_t}{P_{t-1}}\right) = \mu + v_t \varepsilon_t^* + J_t Z_t^y, \quad t = 1, 2, \dots, T \tag{2.1}$$

where

P_t is the future price,

μ is the mean return,

v_t is volatility,

$\varepsilon_t^* = \sqrt{\lambda_t}\varepsilon_t$, where $\varepsilon_t \sim i.i.d\, \mathcal{N}(0,1)$ and $\lambda_t \sim i.i.d\, \mathcal{IG}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$,

J_t is a jump indicator with $P[J_t = 1] = \kappa$, and

Z_t^y is return jumps, where $Z_t^y \sim i.i.d\, \mathcal{N}(\mu_y, \sigma_y^2)$.

Here, $\mathcal{N}$ is the normal distribution and $\mathcal{IG}$ is the inverse gamma distribution. The volatilities, v_t, are expressed as a multiplicative form:

$$v_t = \sigma \cdot X_{t,1} \cdot X_{t,2} \cdot S_t \cdot A_t, \tag{2.2}$$

where $X_{t,1}$ and $X_{t,2}$ are SV (Stochastic Volatility) processes, S_t is seasonal component, and A_t is announcement component. Then, the logarithm of the variance can be rewritten as

$$h_t = \log(v_t^2) = \mu_h + x_{t,1} + x_{t,2} + s_t + a_t, \tag{2.3}$$

where $\mu_h = \log(\sigma^2)$, $x_{t,i} = \log(X_{t,i}^2)$ for $i = 1, 2$, $s_t = \log(S_t^2)$, and $a_t = \log(A_t^2)$.

In what follows, we explain each component in (2.3). The SV processes, $x_{t,1}$ and $x_{t,2}$, are given by

$$x_{t+1,1} = \phi_1 x_{t,1} + \sigma_1 \eta_{t,1}, \tag{2.4}$$

$$x_{t+1,2} = \phi_2 x_{t,2} + \sigma_2\left(\rho\varepsilon_t + \sqrt{1-\rho^2}\,\eta_{t,2}\right) + J_t Z_t^v, \tag{2.5}$$

where

$\eta_{t,i} \sim i.i.d\, \mathcal{N}(0,1)$ for $i = 1, 2$,

$\rho = corr\,(\varepsilon_t, \eta_{t,2})$ represents the correlation between returns and the process $x_{t,2}$,

J_t is the jump time, and

$Z_t^v \sim i.i.d\, \mathcal{N}(\mu_v, \sigma_v^2)$ are the jump size in log volatility.

Note that the jump time in the SV process $x_{t,2}$ corresponds to those in returns. The SV process $x_{t,1}$ explains the persistence of interday volatility. On the other hand, the SV process $x_{t,2}$ represents the short–term impact of high–frequency news or liquidity events. We refer to $x_{t,1}$ and $x_{t,2}$ as “slow” and “fast” volatility factors, respectively, and assume that $0 < \phi_2 < \phi_1 < 1$.

Seasonal components explain the deterministic volatility patterns during trading hours. It is known that the volatility patterns typically have the smooth U–shaped patterns. The seasonal components, s_t, are given by

$$s_t = \sum_{k=1}^{K} H_{tk}\beta_k \tag{2.6}$$

where H_{tk} is an indicator, and β_k is the seasonal effect at period k. The coefficients β_k for seasonal components are estimated using state–space form for cubic smoothing splines.

Announcement components are factors for the effect of macroeconomic announcements and specific events. Stroud and Johannes [7] assumes that it has a short–term impact, an increasing of volatility lasts for 30 minutes. Announcement component is

$$a_t = \sum_{i=1}^{n}\sum_{k=1}^{5} I_{itk}\alpha_{ik}, \tag{2.7}$$

where α_{ik} is the announcement effect for news type i at k period after the news release for $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, 5$, and I_{itk} is an indicator for news type i at period k. The coefficients α_{ik} for announcement components are modeled by a state–space form as with seasonal components.

3 Estimation Procedure

We introduce the estimation method for intraday volatility proposed in Stroud and Johannes [7]. The log–return equation

$$y_t = \mu + \exp\left(\frac{h_t}{2}\right)\sqrt{\lambda_t}\varepsilon_t + J_t Z_t^y \tag{3.1}$$

can be written as

$$y_t^* = h_t + \log(\varepsilon_t^2), \tag{3.2}$$

where

$$y_t^* = \log\left[\frac{(y_t - \mu - J_t Z_t^y)^2}{\lambda_t}\right], \quad d_t = sign(y_t - \mu - J_t Z_t^y), \tag{3.3}$$

and we let $\boldsymbol{y}^* = (y_1^*, \ldots, y_T^*)'$. Following Kim et al. [5] and Omori et al. [6], we set $\zeta_t = \log(\varepsilon_t^2)$ and approximate the distribution of ζ_t by a mixture of normal distributions

$$f(\zeta_t) = \sum_{j=1}^{K} p_j \mathcal{N}(\zeta_t | m_j, v_j^2) \tag{3.4}$$

where $\mathcal{N}(\zeta_t | m_j, v_j^2)$ is the density function of normal distribution with mean m_j and variance v_j^2. The constant values of m_j and v_j are showed in Kim et al. [5] and Omori et al. [6]. The conditional distribution of $\eta_{t,2}$ given d_t, ζ_t and ρ is given by

$$\eta_{t,2} \,|\, d_t, \zeta_t, \rho \sim \mathcal{N}\left(d_t\rho\exp\left(\frac{\zeta_t}{2}\right), 1 - \rho^2\right). \tag{3.5}$$

We then approximate the joint distribution of ζ_t and $\eta_{t,2}$ by a following mixture of normal distributions

$$p(\zeta_t, \eta_{t,2}|d_t, \rho) = \sum_{j=1}^{K} p_j \mathcal{N}(\zeta_t|m_j, v_j^2)\mathcal{N}(\eta_{t,2}|d_t\rho(a_j^* + b_j^*\zeta_t), 1-\rho^2) \tag{3.6}$$

where $(p_j, m_j, v_j, a_j^*, b_j^*)$ are constants shown in Omori et al. [6]. Note that we approximate $\exp(\zeta_t/2)$ in (3.5) by $a_j^* + b_j^*\zeta_t$ in (3.6). We introduce the latent mixture component indicators $\omega_t \in \{1, 2, \ldots, K\}$ and let $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_T)$.

Under the above setup, the posterior density for the states and parameters shown in Section 2 is given by

$$p(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\theta}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\theta})p(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{J}, \boldsymbol{Z}|\boldsymbol{\theta})p(\boldsymbol{\beta}|\boldsymbol{\theta})p(\boldsymbol{\alpha}|\boldsymbol{\theta})p(\boldsymbol{\theta}) \tag{3.7}$$

where $\boldsymbol{x} = (\boldsymbol{x_1}, \ldots, \boldsymbol{x_T})$, $\boldsymbol{x_t} = (x_{t,1}, x_{t,2})$, $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_T)$, $\boldsymbol{J} = (J_1, \ldots, J_T)$, $\boldsymbol{Z} = (\boldsymbol{Z_1}, \ldots, \boldsymbol{Z_T})$, $\boldsymbol{Z_t} = (Z_t^y, Z_t^v)$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_K)'$, $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_n)'$, $\boldsymbol{\alpha}_k = (\alpha_1, \ldots, \alpha_5)$,
$\boldsymbol{\theta} = (\mu, \mu_h, \phi_1, \phi_2, \sigma_1, \sigma_2, \rho, \nu, \kappa, \mu_y, \sigma_y, \mu_v, \sigma_v, \tau_s, \tau_a)$ are parameters, and $\boldsymbol{y} = (y_1, \ldots, y_T)'$ are data of returns.

We first take appropriate initial values of $\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\beta}, \boldsymbol{\alpha}$, and $\boldsymbol{\theta}$. The MCMC algorithm to generate random draws is shown as below.

Step 1 Draw $p(\boldsymbol{\omega}|\boldsymbol{y}^*, \boldsymbol{x}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\lambda}, \boldsymbol{\theta})$

Step 2 Draw $p(\boldsymbol{x}, \mu_h, \phi_i, \sigma_i, \rho|\boldsymbol{y}^*, \boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{\alpha})$, $i = 1, 2$

Step 3 Draw $p(\boldsymbol{\beta}, \tau_s^2|\boldsymbol{y}^*, \boldsymbol{\omega}, \boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{\theta})$

Step 4 Draw $p(\boldsymbol{\alpha}, \tau_a^2|\boldsymbol{y}^*, \boldsymbol{\omega}, \boldsymbol{x}, \boldsymbol{\beta}, \boldsymbol{\theta})$

Step 5 Draw $p(\boldsymbol{\lambda}, \nu|\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\beta}, \boldsymbol{\alpha}, \mu)$

Step 6 Draw $p(\boldsymbol{J}, \boldsymbol{Z}|\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{\lambda}, \kappa, \mu_j, \sigma_j, \mu)$, $j = y, v$

Step 7 Draw $p(\kappa, \mu_j, \sigma_j|\boldsymbol{J}, \boldsymbol{Z})$, $j = y, v$

Step 8 Draw $p(\mu|\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{J}, \boldsymbol{Z}, \boldsymbol{\beta}, \boldsymbol{\alpha})$

3.1 The MCMC Algorithm

We explain the MCMC Algorithm in detail as follows.

3.1.1 Step 1

For sampling of mixture component indicators $\omega_t \in \{1, 2, \ldots, K\}$, we evaluate

$$p(\omega_t = j|\zeta_t, \eta_{t,2}, \rho) \propto p_j \mathcal{N}(\zeta_t|m_j, v_j^2)\mathcal{N}(\eta_{t,2}|d_t\rho(a_j^* + b_j^*\zeta_t), 1-\rho^2) \tag{3.8}$$

for each $j = 1, 2, \ldots, K$. The random draws of ω_t are generated by inverse distribution methods.

3.1.2 Step 2

Given the mixture component indicators ω_t and the other states and parameters, the SV model can be expressed in linear Gaussian state space form:

$$\hat{y}_t = \mu_h + x_{t,1} + x_{t,2} + v_{\omega_t} u_{t,1}, \tag{3.9}$$

$$x_{t+1,1} = \phi_1 x_{t,1} + \sigma_1 u_{t,2}, \tag{3.10}$$

$$x_{t+1,2} = \phi_2 x_{t,2} + \sigma_2 \left(d_t \rho (a^*_{\omega_t} + b^*_{\omega_t} u_{t,1}) + \sqrt{1-\rho^2} u_{t,3} \right) + J_t Z^v_t, \tag{3.11}$$

where $\hat{y}_t = y^*_t - s_t - a_t + m_{\omega_t}$, $(u_{t,1}, u_{t,2}, u_{t,3})' \sim \mathcal{N}_3(\mathbf{0}, \mathbf{I})$, and $\mathcal{N}_p(\mathbf{m}, \mathbf{V})$ denotes a p–variate normal distribution with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{V}$.

(i) Sampling the parameters $\boldsymbol{\psi} = (\phi_1, \phi_2, \sigma_1, \sigma_2, \rho)$. We take the sampling density:

$$g(\boldsymbol{\psi}|\hat{\boldsymbol{y}}, \boldsymbol{d}, \boldsymbol{\omega}) \propto g(\hat{\boldsymbol{y}}|\boldsymbol{d}, \boldsymbol{\omega}, \boldsymbol{\psi}) \pi(\boldsymbol{\psi}), \tag{3.12}$$

where $\hat{\boldsymbol{y}} = (\hat{y}_1, \ldots, \hat{y}_T)'$, $\boldsymbol{d} = (d_1, \ldots, d_T)'$, and $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_T)$. The density $g(\hat{\boldsymbol{y}}|\boldsymbol{d}, \boldsymbol{\omega}, \boldsymbol{\psi})$ is evaluated by the output of the Kalman filter for the state–space model in (3.9)–(3.11). $\pi(\boldsymbol{\psi})$ is a given prior density on $\boldsymbol{\psi}$. Following the approach of Omori et al. [6], we generate the random draws of parameters $\boldsymbol{\psi}$ using a Metropolis–Hastings algorithm with a proposal density based on a truncated multivariate normal distribution.

(ii) Sampling $(x_{t,1}, x_{t,2}, \mu_h)$. We apply a *forwards filtering, backwards sampling* algorithm by means of Software.

3.1.3 Step 3

Let $\boldsymbol{\beta}^*_k = (\beta_k, \dot{\beta}_k)'$ be a state vector. Conditional on the other states and parameters, the state–space model for smoothing splines to estimate seasonal component $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_K)'$ can be expressed as

$$\hat{y}_k = \boldsymbol{h}'\boldsymbol{\beta}^*_k + \varepsilon_k, \quad \varepsilon_k \sim \mathcal{N}(0, \hat{v}^2_k), \tag{3.13}$$

$$\boldsymbol{\beta}^*_{k+1} = \boldsymbol{F}\boldsymbol{\beta}^*_k + \boldsymbol{u}_k, \quad \boldsymbol{u}_k \sim \mathcal{N}(\mathbf{0}, c^2_k \tau^2_s \boldsymbol{U}) \tag{3.14}$$

for $k = 1, 2, \ldots, K$ where

$$\hat{y}_k = \hat{v}^2_k \sum_{\{t:H_{tk}=1\}} (y^*_t - \mu - x_{t,1} - x_{t,2} - a_t - m_{\omega_t}) v^{-2}_{\omega_t}, \tag{3.15}$$

$$\hat{v}^2_k = \left(\sum_{\{t:H_{tk}=1\}} v^{-2}_{\omega_t} \right)^{-1}, \tag{3.16}$$

$$\boldsymbol{h} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{F} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} 1/3 & 1/2 \\ 1/2 & 1 \end{pmatrix}, \tag{3.17}$$

and c_k are a known scale factor for the variance. In this model, $\boldsymbol{\beta}$ and τ_s^2 have to be estimated. We let $\boldsymbol{\beta}^* = (\boldsymbol{\beta}_1^*, \ldots, \boldsymbol{\beta}_K^*)$ and $\hat{\boldsymbol{y}} = (\hat{y}_1, \ldots, \hat{y}_K)'$ and assume a prior density $\tau_s^2 \sim p(\tau_s^2)$. Then, the posterior distribution of $\boldsymbol{\beta}^*$ and τ_s^2 given $\hat{\boldsymbol{y}}$ is

$$p(\boldsymbol{\beta}^*, \tau_s^2 | \hat{\boldsymbol{y}}) \propto p(\tau_s^2) \prod_{k=1}^{K} p(\hat{y}_k | \boldsymbol{\beta}_k^*) p(\boldsymbol{\beta}_{k+1}^* | \boldsymbol{\beta}_k^*, \tau_s^2). \tag{3.18}$$

We generate $\boldsymbol{\beta}^*$ and τ_s^2 from the distribution of (3.18) using a Metropolis–Hastings algorithm in random walk chain. Conditional on the current value, $\tau_s^{2(i)}$, we draw $\tau_s^{2(*)}$ by $\tau_s^{2(*)} \sim \mathcal{N}(\tau_s^{2(i)}, w)$ and accept with probability

$$\min\left\{1, \frac{(\hat{\boldsymbol{y}}|\tau_s^{2(*)}) p(\tau_s^{2(*)})}{(\hat{\boldsymbol{y}}|\tau_s^{2(i)}) p(\tau_s^{2(i)})}\right\} \tag{3.19}$$

where $p(\hat{\boldsymbol{y}}|\tau_s^2)$ are computed by the Kalman filter. If the random draw is accepted, we set $\tau_s^{2(i+1)} = \tau_s^{2(*)}$ and generate $\boldsymbol{\beta}^{*(i+1)} \sim p(\boldsymbol{\beta}^* | \tau_s^{2(i+1)}, \hat{\boldsymbol{y}})$ using a *forwards filtering, backwards sampling* algorithm. Otherwise set $\tau_s^{2(i+1)} = \tau_s^{2(i)}$ and leave $\boldsymbol{\beta}^*$ unchanged. The desired random draws $\boldsymbol{\beta}$ are obtained from $\boldsymbol{\beta}^*$.

3.1.4 Step 4

We define a state vector $\boldsymbol{\alpha}_{i,k}^* = (\alpha_{i,k}, \dot{\alpha}_{i,k})'$. Then, the state–space model for cubic smoothing splines to estimate announce components can be obtained as

$$\hat{y}_{ik} = \boldsymbol{h}' \boldsymbol{\alpha}_{i,k}^* + \varepsilon_{i,k}, \quad \varepsilon_{i,k} \sim \mathcal{N}(0, \hat{v}_{i,k}^2) \tag{3.20}$$

$$\boldsymbol{\alpha}_{i,k+1}^* = \boldsymbol{F} \boldsymbol{\alpha}_{i,k}^* + \boldsymbol{u}_{i,k}, \quad \boldsymbol{u}_{i,k} \sim \mathcal{N}(\boldsymbol{0}, \tau_a^2 \boldsymbol{U}) \tag{3.21}$$

for $i = 1, \ldots, n$ and $k = 1, \ldots, 5$ where

$$\hat{y}_{ik} = \hat{v}_{ik}^2 \sum_{\{t: I_{tik}=1\}} (y_t^* - \mu - x_{t,1} - x_{t,2} - s_t - m_{\omega_t}) v_{\omega_t}^{-2}, \tag{3.22}$$

$$\hat{v}_{ik}^2 = \left(\sum_{\{t: I_{tik}=1\}} v_{\omega_t}^{-2} \right)^{-1}. \tag{3.23}$$

The Metropolis–Hastings algorithm is implemented to generate the random draws of $\boldsymbol{\alpha}$ and τ_a in the same way as Step 3.

3.1.5 Step 5

The joint posterior of $\boldsymbol{\lambda}$ and ν is $p(\boldsymbol{\lambda}, \nu | rest) = p(\nu | rest) p(\boldsymbol{\lambda} | \nu, rest)$ where "*rest*" represents the other states and parameters.

(i) Sampling the degrees of freedom ν. We define $w_t = (y_t - \mu - J_t Z_t^y)/V_t$. It follows that $(w_t | \nu, rest) \sim t_\nu$ where t_ν denotes the Student–t density with ν degrees of freedom. Under a prior $\nu \sim \mathcal{DU}(2, 128)$ where $\mathcal{DU}$ is discrete uniform distribution, the posterior density is obtained as a multinomial distribution $(\nu | w, rest) \sim \mathcal{M}(\pi_2^*, \ldots, \pi_{128}^*)$ with probabilities:

$$\pi_\nu^* \propto \prod_{t=1}^{T} p_\nu(w_t) \tag{3.24}$$

for $\nu = 2, \ldots, 128$ where $p_\nu(\cdot)$ represents the Student–t density with ν degrees of freedom. We use a Metropolis–Hastings algorithm to sample ν. Conditional on the current value, $\nu^{(i)}$, we generate a candidate value $\nu^{(*)}$ from $\nu^{(*)} \sim \mathcal{DU}(\nu^{(i)} - \delta, \nu^{(i)} + \delta)$ and accept with probability

$$\min\left\{1, \frac{\prod_{t=1}^{T} p_{\nu^{(*)}}(w_t)}{\prod_{t=1}^{T} p_{\nu^{(i)}}(w_t)}\right\} \tag{3.25}$$

where the width δ is chosen to achieve an acceptance probability between 20% and 50%.

(ii) Sampling the scale factors $\boldsymbol{\lambda}$. We define $\varepsilon_t^* = (y_t - \mu - J_t Z_t^y)/\sqrt{V_t}$. Under $(\varepsilon_t^* | \lambda_t, \nu, rest) \sim \mathcal{N}(0, \lambda_t)$ and $(\lambda_t | \nu) \sim \mathcal{IG}(\nu/2, \nu/2)$ where $\mathcal{IG}$ is the inverse gamma distribution, the full conditional is

$$(\lambda_t | \nu, rest) \sim \mathcal{IG}\left(\frac{\nu+1}{2}, \frac{\nu + \varepsilon_t^{*2}}{2}\right). \tag{3.26}$$

3.1.6 Step 6

By arranging the log–return (2.1) and the fast–volatility (3.11), the model given the other states and parameters is obtained as $(\boldsymbol{w}_t | J_t, \boldsymbol{Z}_t, rest) \sim \mathcal{N}(J_t \boldsymbol{Z}_t, \boldsymbol{\Sigma}_t)$, where

$$\boldsymbol{w}_t = \begin{pmatrix} y_t - \mu \\ x_{t+1,2} - \phi_2 x_{t,2} \end{pmatrix}, \quad \boldsymbol{Z}_t = \begin{pmatrix} Z_t^y \\ Z_t^v \end{pmatrix}, \quad \boldsymbol{\Sigma}_t = \begin{pmatrix} \lambda_t V_t & \rho\sigma_2\sqrt{\lambda_t V_t} \\ \rho\sigma_2\sqrt{\lambda_t V_t} & (1-\rho^2)\sigma_2^2 \end{pmatrix}. \tag{3.27}$$

We assume the conjugate priors $J_t \sim Bern(\kappa)$ and $\boldsymbol{Z}_t \sim \mathcal{N}(\boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z)$, where $Bern$ is the Bernoulli distribution, $\boldsymbol{\mu}_z = (\mu_y, \mu_v)'$ and $\boldsymbol{\Sigma}_z = \mathrm{diag}(\sigma_y^2, \sigma_v^2)$. Then the full conditionals for the jump times, J_t, and jump sizes, $\boldsymbol{Z}_t$ are given by

$$P(J_t = 1 | rest) = \frac{\kappa\, \phi(\boldsymbol{w}_t; \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_t + \boldsymbol{\Sigma}_z)}{(1-\kappa)\phi(\boldsymbol{w}_t; \boldsymbol{0}, \boldsymbol{\Sigma}_t) + \kappa\, \phi(\boldsymbol{w}_t; \boldsymbol{\mu}_z, \boldsymbol{\Sigma}_t + \boldsymbol{\Sigma}_z)},$$

$$(\boldsymbol{Z}_t | J_t = 1, rest) \sim \mathcal{N}\left((\boldsymbol{\Sigma}_z^{-1} + \boldsymbol{\Sigma}_t^{-1})^{-1}(\boldsymbol{\Sigma}_z^{-1}\boldsymbol{\mu}_z + \boldsymbol{\Sigma}_t^{-1}\boldsymbol{w}_t), (\boldsymbol{\Sigma}_z^{-1} + \boldsymbol{\Sigma}_t^{-1})^{-1}\right).$$

3.1.7 Step 7

We assume the conjugate priors $\kappa \sim \mathcal{B}(a_k, b_k)$ and $(\mu_j, \sigma_j^2) \sim \mathcal{NIG}(m_j, c_j, a_j, b_j)$ for $j = y, v$, where $\mathcal{B}$ is the beta distribution and $\mathcal{NIG}$ is the normal–inverse gamma distribution. Then, the full conditionals are given by

$$(\kappa | rest) \sim \mathcal{B}(a_k^*, b_k^*), \tag{3.28}$$

$$(\mu_y, \sigma_y^2 | rest) \sim \mathcal{NIG}(m_y^*, c_y^*, a_y^*, b_y^*), \tag{3.29}$$

$$(\mu_v, \sigma_v^2 | rest) \sim \mathcal{NIG}(m_v^*, c_v^*, a_v^*, b_v^*), \tag{3.30}$$

where

$$a_k^* = a_k + \sum_{t=1}^{T} J_t, \quad b_k^* = b_k + T - \sum_{t=1}^{T} J_t, \tag{3.31}$$

$$c_j^* = c_j + \sum_{t=1}^{T} J_t, \quad c_j^* m_j^* = c_j m_j + \sum_{t=1}^{T} J_t Z_t^j, \tag{3.32}$$

$$a_j^* = a_j + \sum_{t=1}^{T} J_t, \quad b_j^* = b_j + c_j m_j^2 + \sum_{t=1}^{T} (J_t Z_t^j)^2 - c_j^* m_j^{*2} \tag{3.33}$$

for $j = y, v$.

3.1.8 Step 8

Under the prior density $\mu \sim \mathcal{N}(m_\mu, v_\mu)$, the full conditional is given by

$$(\mu | rest) \sim \mathcal{N}\left(\left(\frac{1}{v_\mu} + \sum_{t=1}^{T} \frac{1}{V_t^*}\right)^{-1} \left(\frac{m_\mu}{v_\mu} + \sum_{t=1}^{T} \frac{y_t^*}{V_t^*}\right), \left(\frac{1}{v_\mu} + \sum_{t=1}^{T} \frac{1}{V_t^*}\right)^{-1}\right), \tag{3.34}$$

where

$$V_t^* = (1 - \rho^2)\lambda_t V_t \tag{3.35}$$

$$y_t^* = y_t - J_t Z_t^y - \frac{\rho\sqrt{\lambda_t V_t}}{\sigma_2}(x_{t+1,2} - \phi_2 x_{t,2} - J_t Z_t^v). \tag{3.36}$$

4 Conclusion and the future works

We have discussed a procedure to analyze volatilities of high–frequency returns based on the framework of Bayesian modeling. The intraday volatilities have been described as the product of four components—slow volatility, fast volatility, seasonal effect, and announcement effect. Each factor of volatility has been sampled through Markov Chain Monte Carlo (MCMC) methods using a state–space form.

In the next step, we would apply the estimation procedure to data of 5–min return for Nikkei 225 Futures. Based on the analyzed result, we would like to propose a new approach to forecast volatilities in Japan.

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Graduate School of Economics
Osaka University, Toyonaka 560-0043, Japan
E-mail address: n-ochiai@econ.osaka-u.ac.jp

大阪大学・経済学研究科　落合夏海

Graduate School of Economics
Osaka University, Toyonaka 560-0043, Japan
E-mail address: ohnishi@econ.osaka-u.ac.jp

大阪大学・経済学研究科　大西匡光